Discrete controller Design

Deadbeat, Dahlin Controller and Pole-Placement Control

Digital control

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Introduction

- The design of a digital control system begins with an **accurate model of the process** to be controlled.
- Then a control algorithm is developed that will give the required system response.
- The loop is closed by using a digital computer as the controller.
- The computer implements the control algorithm in order to achieve the required response.

Introduction

- **Several methods can be used for the design of a digital controller:**

- A system transfer function is modelled and obtained in the *s*-plane. The transfer function is then transformed into the *z*-plane and the controller is designed in the *z*-plane.

- System transfer function is modelled as a digital system and the controller is directly designed in the *z*-plane.

- The continuous system transfer function is transformed into the *w*-plane. A suitable controller is then designed in the *w*-plane using the well-established time response (e.g. root locus) or frequency response (e.g. Bode diagram) techniques.
- The final design is transformed into the *z*-plane and the algorithm is implemented on the digital computer.

Introduction

- We are mainly interested in the design of a digital controller using the first method, i.e. **the controller is designed directly in the *z*-plane.**

- **The procedure for designing the controller in the *z*-plane can be outlined as follows:**

1. Derive the transfer function of the system either by using a mathematical approach or by performing a frequency or a time response analysis.
2. Transform the system transfer function into the *z*-plane.
3. Design a suitable digital controller in the *z*-plane.
4. Implement the controller algorithm on a digital computer.

Introduction

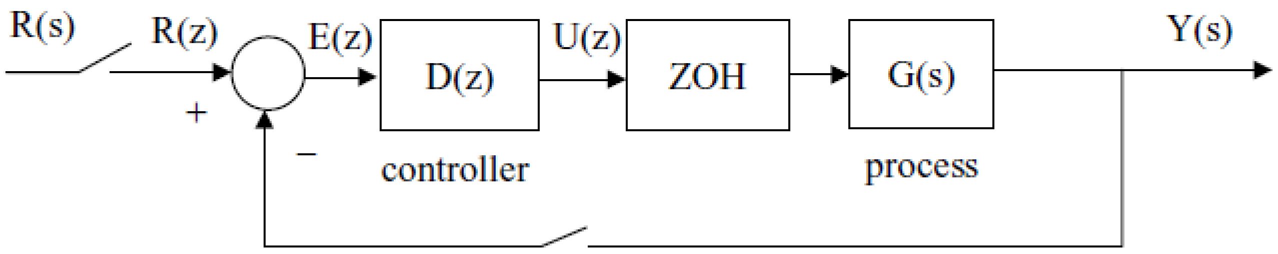

Figure 9.1 Discrete-time system with analog reference input

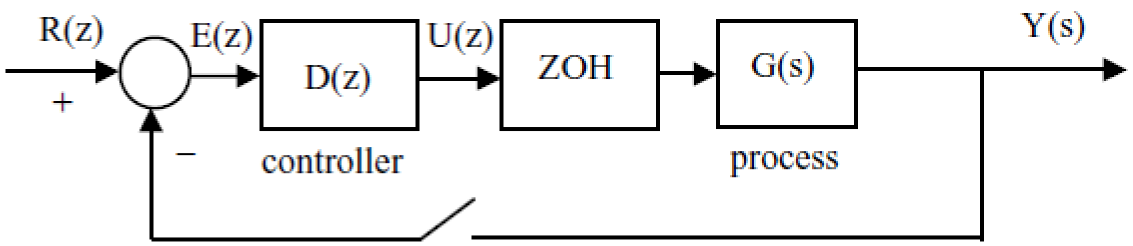

Figure 9.2 Discrete-time system with digital reference input

Introduction

The closed-loop transfer function of the system in Figure 9.3 can be written as

$$\frac{Y(z)}{R(z)} = \frac{D(z)HG(z)}{1 + D(z)HG(z)}. \tag{9.1}$$

Now, suppose that we wish the closed-loop transfer function to be $T(z)$, i.e.

$$T(z) = \frac{Y(z)}{R(z)}. \tag{9.2}$$

Then the required controller that will give this closed-loop response can be found by using (9.1) and (9.2):

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)}. \tag{9.3}$$

$R(z)$ is the reference input, $E(z)$ is the error signal, $U(z)$ is the output of the controller, and $Y(z)$ is the output of the system. $HG(z)$ represents the digitized plant transfer function together with the zero-order hold.

Introduction

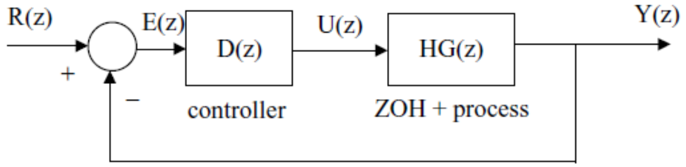

Figure 9.3 Discrete-time system

The closed-loop transfer function of the system in Figure 9.3 can be written as

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$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)}. \tag{9.3}$$

Equation (9.3) states that the required controller $D(z)$ can be designed if we know the model of the process. The controller $D(z)$ must be chosen so that it is stable and can be realized. One of the restrictions affecting realizability is that $D(z)$ must not have a numerator whose order
exceeds that of the denominator.

Dead-Beat Controller

The dead-beat controller is one in which a step input is followed by the system but delayed by one or more sampling periods, i.e. the system response is required to be equal to unity at every sampling instant after the application of a unit step input.

The required closed-loop transfer function is then

$$T(z) = z^{-k}, \qquad \text{where } k \geq 1. \tag{9.4}$$

From (9.3), the required digital controller transfer function is

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)} = \frac{1}{HG(z)} \left(\frac{z^{-k}}{1 - z^{-k}} \right). \tag{9.5}$$

An example design of a controller using the dead-beat algorithm is given below.

Dead-Beat Controller

Example 9.1

The open-loop transfer function of a plant is given by

$$G(s) = \frac{e^{-2s}}{1 + 10s}.$$

Design a dead-beat digital controller for the system. Assume that $T = 1$ s.

Solution

The transfer function of the system with a zero-order hold is given by

$$HG(z) = Z\left\{\frac{1 - e^{-sT}}{s} G(s)\right\} = (1 - z^{-1})Z\left\{\frac{e^{-2s}}{s(1 + 10s)}\right\}$$

or

$$HG(z) = (1 - z^{-1})z^{-2}Z\left\{\frac{1}{s(1 + 10s)}\right\} = (1 - z^{-1})z^{-2}Z\left\{\frac{1/10}{s(s + 1/10)}\right\}.$$

From z-transform tables we obtain

$$HG(z) = (1 - z^{-1})z^{-2}\frac{z(1 - e^{-0.1})}{(z - 1)(z - e^{-0.1})} = z^{-3}\frac{(1 - e^{-0.1})}{1 - e^{-0.1}z^{-1}}$$

or

$$HG(z) = \frac{0.095z^{-3}}{1 - 0.904z^{-1}}.$$

Dead-Beat Controller

From Equations (9.3) and (9.5),

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)} = \frac{1 - 0.904z^{-1}}{0.095z^{-3}} \frac{z^{-k}}{1 - z^{-k}}.$$

For realizability, we can choose $k \geq 3$. Choosing $k = 3$, we obtain

$$D(z) = \frac{1 - 0.904z^{-1}}{0.095z^{-3}} \frac{z^{-3}}{1 - z^{-3}}$$

or

$$D(z) = \frac{z^3 - 0.904z^2}{0.095(z^3 - 1)}.$$

Dead-Beat Controller

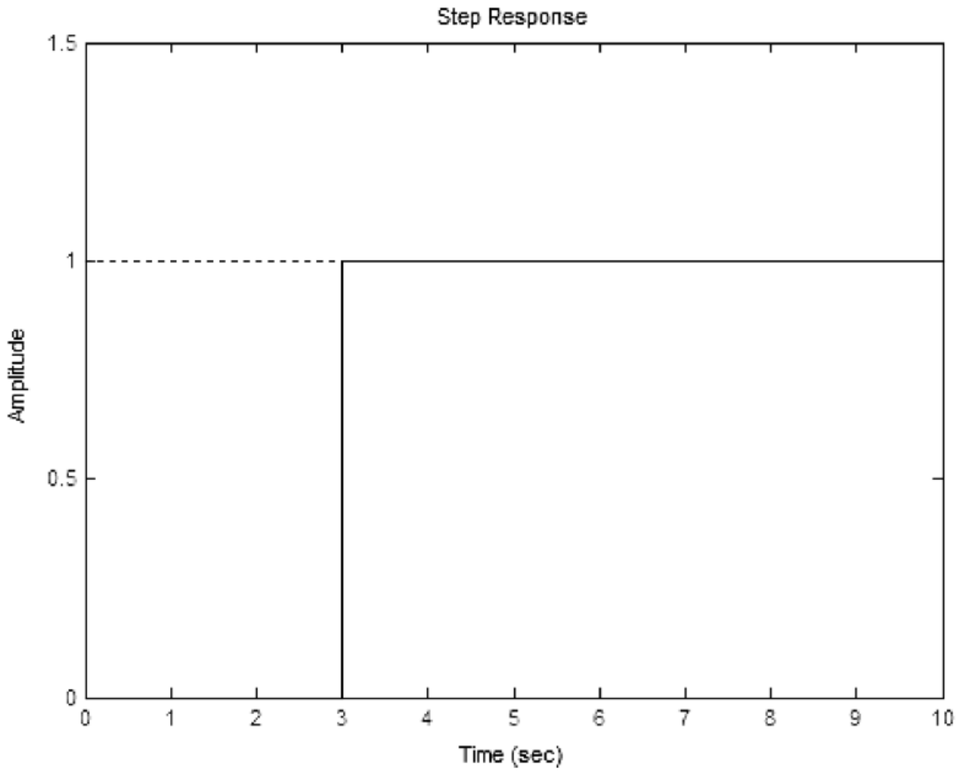

Figure 9.5 Step response of the system

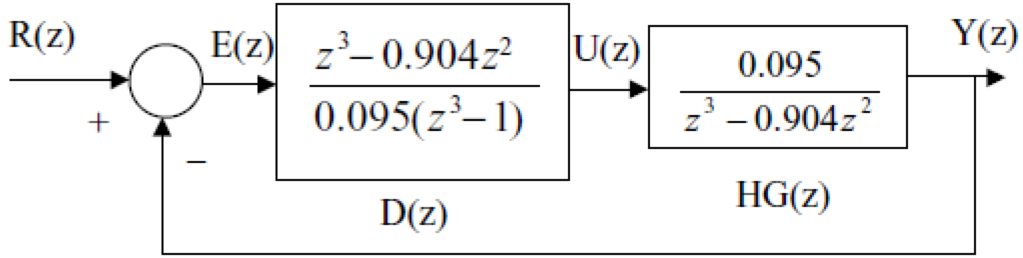

Figure 9.4 Block diagram of the system of Example 9.1

The output response is unity after 3 s (third sample) and stays at this value

Dead-Beat Controller

It is important to realize that the response is correct only at the sampling instants and the response can have an oscillatory behaviour between the sampling instants.

The control signal applied to the plant is shown in Figure 9.6. Although the dead-beat controller has provided an excellent response, the magnitude of the control signal may not be acceptable, and it may even saturate in practice.

The dead-beat controller is very sensitive to plant characteristics and a small change in the plant may lead to ringing or oscillatory response.

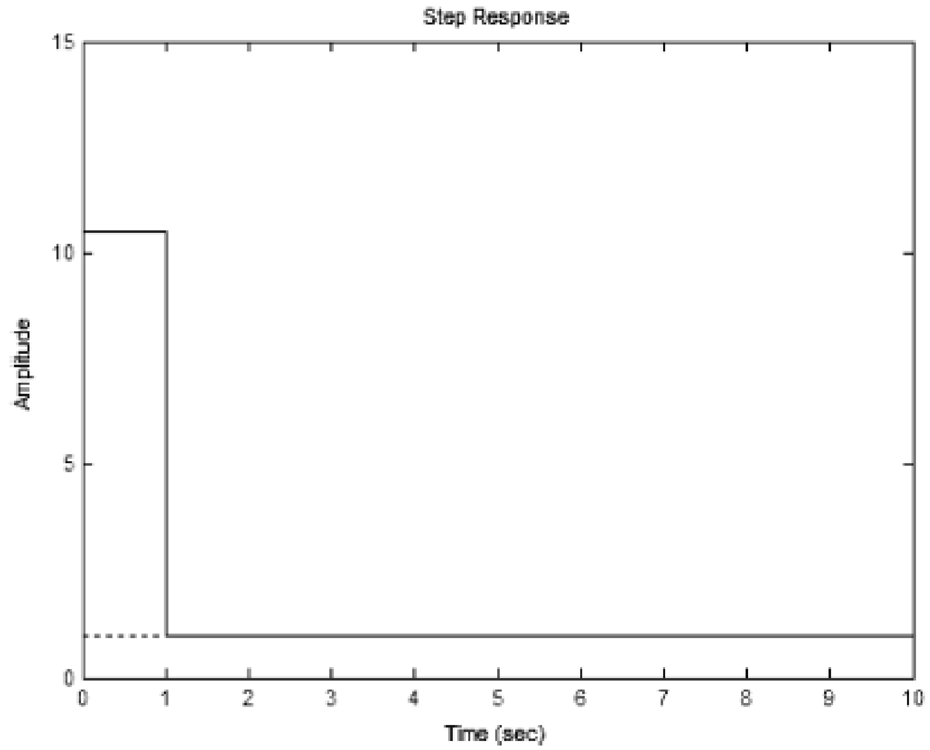

Figure 9.6 Control signal

Dahlin controller

+ The Dahlin controller is a modification of the dead-beat controller and produces an exponential response which is smoother than that of the dead-beat controller.
+ The required response of the system in the *s*-plane can be shown to be

$$Y(s) = \frac{1}{s}\frac{e^{-as}}{1+sq},$$

where a and q are chosen to give the required response (see Figure 9.7). If we let $a = kT$, then the z-transform of the output is

$$Y(z) = \frac{z^{-k-1}(1-e^{-T/q})}{(1-z^{-1})(1-e^{-T/q}z^{-1})}$$

and the required transfer function is

$$T(z) = \frac{Y(z)}{R(z)} = \frac{z^{-k-1}(1-e^{-T/q})}{(1-z^{-1})(1-e^{-T/q}z^{-1})}\frac{(1-z^{-1})}{1}$$

Dahlin controller

or

$$T(z) = \frac{z^{-k-1}(1 - e^{-T/q})}{1 - e^{-T/q} z^{-1}}.$$

Using (9.3), we can find the transfer function of the required controller:

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)} = \frac{1}{HG(Z)} \frac{z^{-k-1}(1 - e^{-T/q})}{1 - e^{-T/q} z^{-1} - (1 - e^{-T/q}) z^{-k-1}}.$$

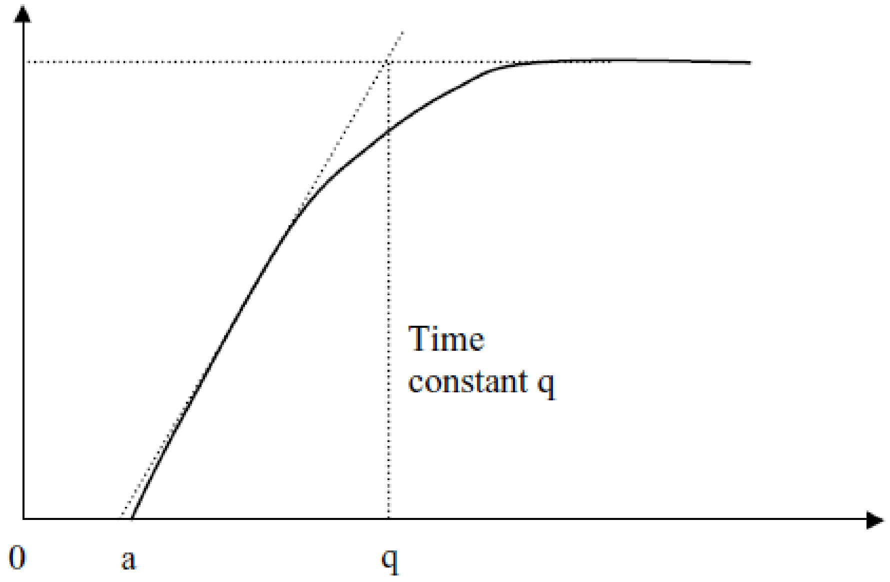

Figure 9.7 Dahlin controller response

Dahlin controller: example

Example 9.2

The open-loop transfer function of a plant is given by

$$G(s) = \frac{e^{-2s}}{1 + 10s}.$$

Design a Dahlin digital controller for the system. Assume that $T = 1$ s.

Solution

The transfer function of the system with a zero-order hold is given by

$$HG(z) = Z\left\{\frac{1 - e^{-sT}}{s} G(s)\right\} = (1 - z^{-1})Z\left\{\frac{e^{-2s}}{s(1 + 10s)}\right\}$$

or

$$HG(z) = (1 - z^{-1})z^{-2}Z\left\{\frac{1}{s(1 + 10s)}\right\} = (1 - z^{-1})z^{-2}Z\left\{\frac{1/10}{s(s + 1/10)}\right\}.$$

From z-transform tables we obtain

$$HG(z) = (1 - z^{-1})z^{-2}\frac{z(1 - e^{-0.1})}{(z - 1)(z - e^{-0.1})} = z^{-3}\frac{(1 - e^{-0.1})}{1 - e^{-0.1}z^{-1}}$$

or

$$HG(z) = \frac{0.095z^{-3}}{1 - 0.904z^{-1}}.$$

Dahlin controller: example

For the controller, if we choose $q = 10$, then

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)} = \frac{1 - 0.904z^{-1}}{0.095z^{-3}} \frac{z^{-k-1}(1 - e^{-0.1})}{1 - e^{-0.1}z^{-1} - (1 - e^{-0.1})z^{-k-1}}$$

or

$$D(z) = \frac{1 - 0.904z^{-1}}{0.095z^{-3}} \frac{0.095z^{-k-1}}{1 - 0.904z^{-1} - 0.095z^{-k-1}}$$

For realizability, if we choose $k = 2$, we obtain

$$D(z) = \frac{0.095z^3 - 0.0858z^2}{0.095z^3 - 0.0858z^2 - 0.0090}.$$

Figure 9.8 shows the step response of the system. It is clear that the response is exponential as expected.

Dahlin controller: example

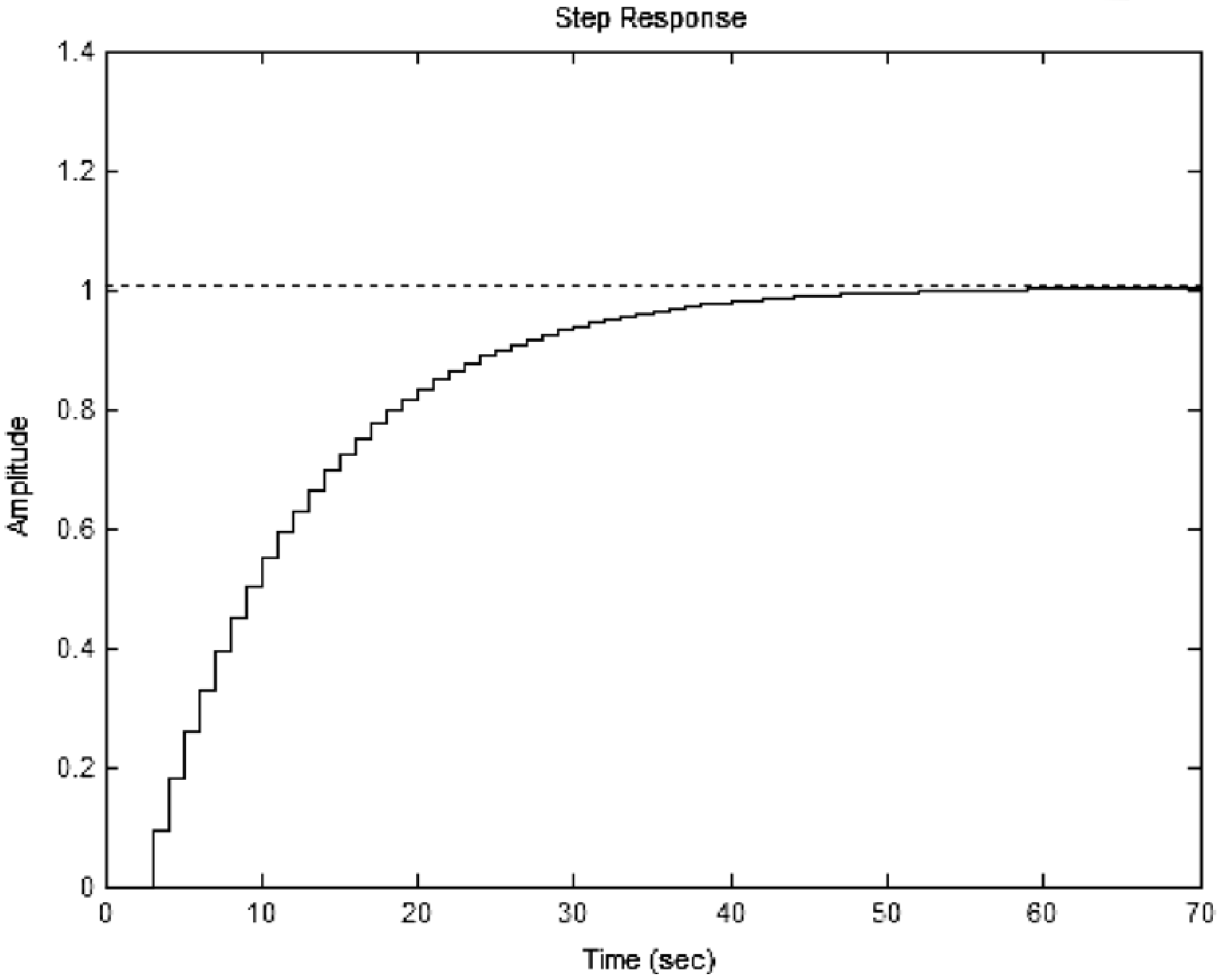

Figure 9.8 System response with Dahlin controller

Dahlin controller: example

Although the system response is Slower than dead-beat controller, the controller signal is more acceptable.

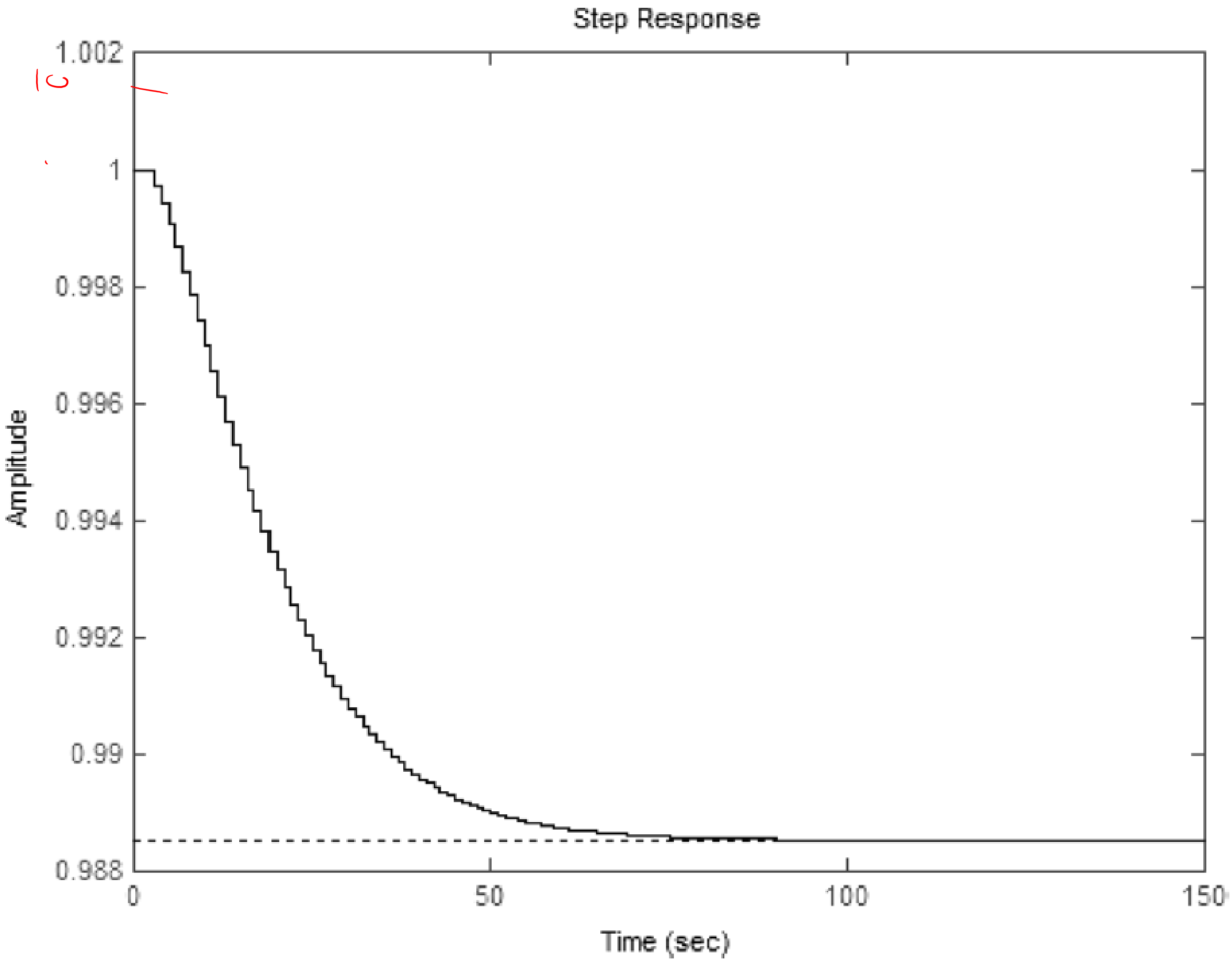

Figure 9.9 Controller response

Pole-Placement Control – Analytical

- The response of a system is determined by the positions of its closed-loop poles. Thus, by placing the poles at the required points we should be able to control the response of a system.

Given the pole positions of a system, (9.3) gives the required transfer function of the controller as

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)}.$$

$T(z)$ is the required transfer function, which is normally in the form of a polynomial.
The denominator of $T(z)$ is constructed from the positions of the required roots.
The numerator polynomial can then be selected to satisfy certain criteria in the system.

Pole-Placement Control: example

Example 9.3

The open-loop transfer function of a system together with a zero-order hold is given by

$$HG(z) = \frac{0.03(z + 0.75)}{z^2 - 1.5z + 0.5}.$$

Design a digital controller so that the closed-loop system will have $\zeta = 0.6$ and $w_d = 3$ rad/s. The steady-state error to a step input should be zero. Also, the steady-state error to a ramp input should be 0.2. Assume that $T = 0.2$ s.

Solution

The roots of a second-order system are given by

$$z_{1,2} = e^{-\zeta\omega_n T \pm j\omega_n T\sqrt{1-\zeta^2}} = e^{-\zeta w_n T}(\cos \omega_n T\sqrt{1-\zeta^2} \pm j \sin \omega_n T\sqrt{1-\zeta^2}).$$

Pole-Placement Control: example

Thus, the required pole positions are

$$z_{1,2} = e^{-0.6\times 3.75\times 0.2}(\cos(0.2\times 3) \pm j\sin(0.2\times 3)) = 0.526 \pm j0.360.$$

The required controller then has the transfer function

$$T(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \ldots}{(z - 0.526 + j0.360)(z - 0.526 - j0.360)}$$

which gives

$$T(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + \ldots}{1 - 1.052z^{-1} + 0.405z^{-2}}. \tag{9.6}$$

We now have to determine the parameters of the numerator polynomial. To ensure realizability, $b_0 = 0$ and the numerator must only have the b_1 and $b_2$terms. Equation (9.6) then becomes

$$T(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 - 1.052z^{-1} + 0.405z^{-2}}. \tag{9.7}$$

The other parameters can be determined from the steady-state requirements.

The steady-state error is given by

$$E(z) = R(z)[1 - T(z)].$$

Pole-Placement Control: example

For a unit step input, the steady-state error can be determined from the final value theorem, i.e.

$$E_{ss} = \lim_{z \to 1} \frac{z-1}{z} \frac{z}{z-1} [v]$$

or

$$E_{ss} = 1 - T(1). \tag{9.8}$$

From (9.8), for a zero steady-state error to a step input,

$$T(1) = 1$$

From (9.7), we have

$$T(1) = \frac{b_1 + b_2}{0.353} = 1$$

or

$$b_1 + b_2 = 0.353, \tag{9.9}$$

and

$$T(z) = \frac{b_1 z + b_2}{z^2 - 1.052z + 0.405}. \tag{9.10}$$

Pole-Placement Control: example

If K_v is the system velocity constant, for a steady-state error to a ramp input we can write

$$E_{ss} = \lim_{z \to 1} \frac{(z-1)}{z} \frac{Tz}{(z-1)^2}[1 - T(z)] = \frac{1}{K_v}$$

or, using L'Hospital's rule,

$$\left.\frac{dT}{dz}\right|_{z=1} = -\frac{1}{K_v T}.$$

Pole-Placement Control: example

Thus from (9.10),

$$\left.\frac{dT}{dz}\right|_{z=1} = \frac{b_1(z^2 - 1.052z + 0.405) - (b_1 z + b_2)(2z - 1.052)}{(z^2 - 1.052z + 0.405)^2} = -\frac{1}{K_v T} = -\frac{0.2}{0.2} = -1,$$

giving

$$\frac{0.353b_1 - (b_1 + b_2)0.948}{0.353^2} = -1$$

or

$$0.595b_1 + 0.948b_2 = 0.124, \tag{9.11}$$

From (9.9) and (9.11) we obtain,

$$b_1 = 0.596 \text{ and } b_2 = -0.243.$$

Equation (9.10) then becomes

$$T(z) = \frac{0.596z - 0.243}{z^2 - 1.052z + 0.405}. \tag{9.12}$$

Pole-Placement Control: example

Equation (9.12) is the required transfer function. We can substitute in Equation (9.3) to find the transfer function of the controller:

$$D(z) = \frac{1}{HG(z)} \frac{T(z)}{1 - T(z)} = \frac{z^2 - 1.5z + 0.5}{0.03(z + 0.75)} \frac{T(z)}{1 - T(z)}$$

or,

$$D(z) = \frac{z^2 - 1.5z + 0.5}{0.03(z + 0.75)} \frac{0.596z - 0.243}{z^2 - 1.648z + 0.648}$$

which can be written as

$$D(z) = \frac{0.596z^3 - 1.137z^2 + 0.662z - 0.121}{0.03z^3 - 0.027z^2 - 0.018z + 0.015} \tag{9.13}$$

The step response of the system with the controller is shown in Figure 9.10.

Pole-Placement Control: example

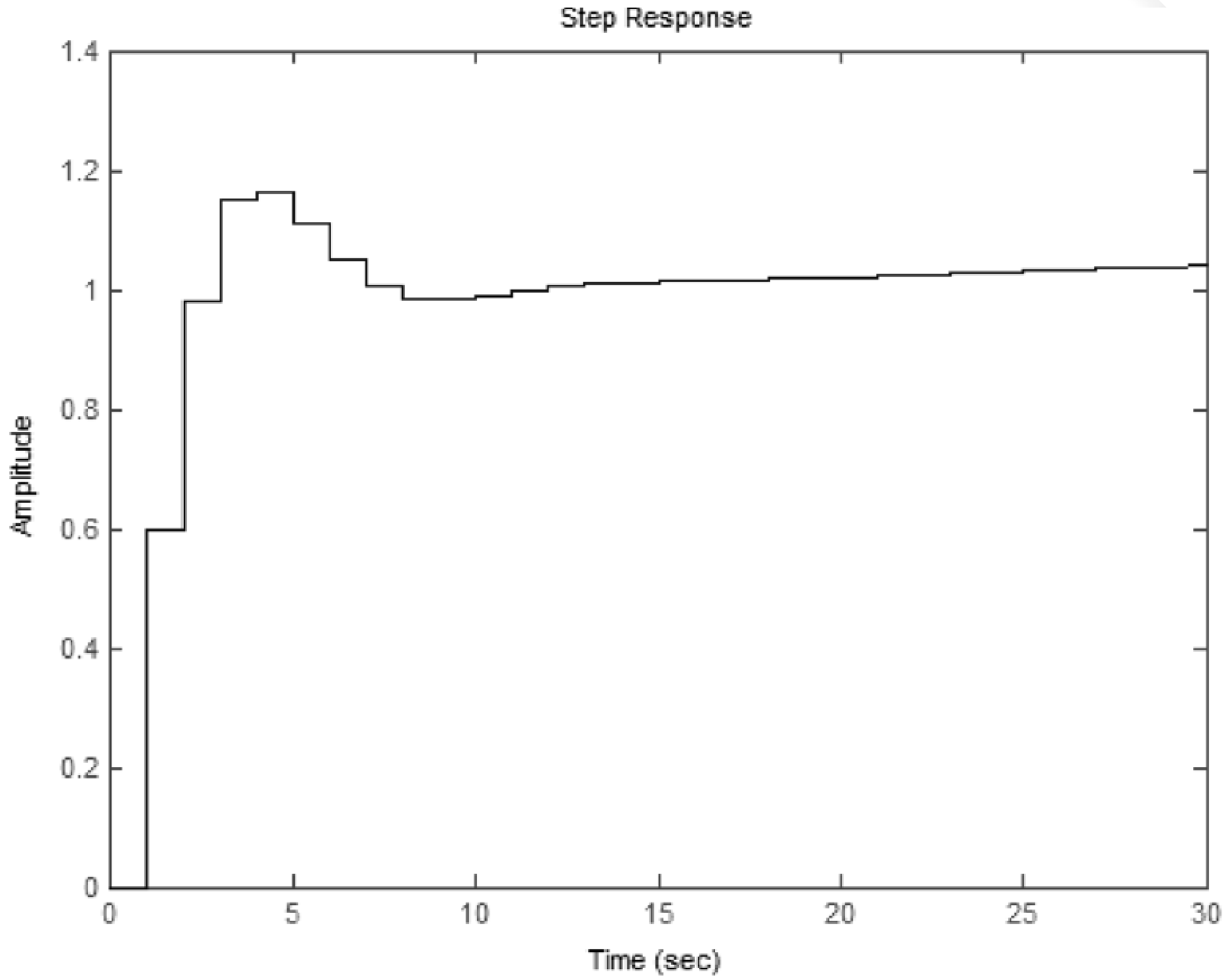

Figure 9.10 Step response of the system